Financial and Actuarial Mathematics

Syllabus for a Master Course

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1. **General goal of the course**

The course provides students with an advanced academic education in Mathematical Finance and Actuarial Science. The students will be able to apply it in practice and will have a base for continuing their education in this field.

Students will be able to solve the basic problems in Financial and Actuarial fields. They will be able to integrate Actuarial, Statistical and Financial techniques in modeling and to take the right decisions in the area of insurance and financial practice.

The course gives a base for the students interested in a furhter PhD degree in Mathematical Modelling and Applications in Economics, Finance and Insurance.

2. Overview on the course modules
PART I: Insurance Risk Models (30 units, 8 credits)

Module	No. of units	Contents
1. Insurance risk	3	The classical model of Insurance risk. Cramer-Lundberg's theorem and inequality. Exercises: Poisson and renewal processes – key renewal theorem.
2. Ruin probability	5	Proof of Cramer – Lundberg's theorem.Pollaczeck-Khinchine formula for the survival probability. Exercises: Ruin probability for exponential claims.
3. Renewal Model	5	Probability of ruin for the renewal model – Random walk approach, ladder points. Andersen's theorem. Wiener – Hopf factorization.
4. The heavy tail case	4	Probability of ruin in the heavy tail case.. Adjustment coefficient. Ruin when both claim sizes and inter-arrival times are heavy tailed. Exercises: Subexponential distributions.
5. Approximation of the risk process	4	Weak convergence of random processes. Diffusion approximation. Approximation with stable Levy motion. Approximation with self-similar processes.
6. Extreme values	5	The model of maxima of independent random variables. Limit distributions for normalized maxima. Extremal processes – a point process approach.

7. Risk measures	4	Lower and upper bounds for ruin probability. Coherent risk measures. VaR – a measure of extreme risk.

PART II: Mathematical Finance (30 units, 8 credites)

8. Stochastic processes	4	Definitions. Examples. Martingales in discrete and continuous time. Local Martingales. The Optional Stopping Theorem. Martingale Convergence.
9. Stochastic processes with independent increments.	5	Random walk. Brownian Motion. First Passage Times. Properties. Stochastic counting processes. Poisson process. Mixed Poisson process.Polya process. Compound Poisson process. Polya-Aeppli process.
10. The Stochastic Integral.	4	Predictable processes. Square-integrable Martingales. Martingales of Bounded Variation. Integration with respect to local martingales.
11. Stochastic Calculus and Ito's formula.	4	Brownian Martingales. Exponential Processes. Change of measure and Girsanov's Theorem. Martingale Representation Theorem.
12. Applications. Introduction in fixed income securities topic	2	Debt instruments: Zero coupon bonds. Corporate bonds. T-bills. Duration. Convexity
13. Term Structure of Interest Rates	3	Models: Nelson - Siegel, Vasicek
14. No Arbitrage Pricing and Risk Neutral Probabilities	4	Financial derivatives (Options, Futures, Swaps). Hedging Interest Rate Risk.
15. Capital Asset Pricing Model	4	Real data analysis
Total no. of units	60	

3. References:

1. Asmussen S. (2000). Ruin Probabilities, World Scientific Publishing Co.
2. Grandell J. (1991) Aspects of Risk Theory, Springer.
3. Grandell J. (1997) Mixed Poisson Processes. Chapman & Hall.
4. Kaas R., Goovaerts M., Dhaene J. and Denuit M. ((2001) Modern Actuarial Risk Theory, Kluwer Academic Publishers.
5. Mikosch Th. (2004) Non–Life Insurance Mathematics, Springer.
6. Elliott R.J. and Kopp P.E. (1999) Mathematics of Financial Markets, Springer.
7. Etheridge A. (2002) A Course in Financial Calculus, Cambridge University Press.
8. Karatzas I. and Shreve S. (1998) Brownian Motion and Stochastic Calculus, Springer.
9. Musiela M. and Rutkowski M. (1997) Martingale Methods in Financial Modelling, Springer.
10. Oksendal B. (1998) Stochastic Differential Equations, 5th edition, Springer.
11. Protter Ph.E. (2004) Stochastic Integration and Differential Equations, 2-nd edition, Springer.
12. Lionel Martellini, Philippe Priaulet, and Stphane Priaulet, 2003, Fixed-Income Securities: Valuation, Risk Management and Portfolio Strategies, The Wiley Finance Series.
13. J. Hull, Options, Futures and other derivatives

4. Teaching

The course should be accompanied by homework exercises. The students can work on their completion during at most 2 of the afternoon sessions. The major part of the afternoon sessions should be spent by working independently in teams. The results also should be presented in the afternoon sessions. During the afternoon sessions the lecturer should be available for questions and be present in order to get an impression on the performance of the students.

The course is planned to last 4 weeks, with lectures from Monday to Friday. This implies that there will be 3 teaching units (45 minutes each) per day. The following schedule is proposed for each day:

8:00 till 11:00: three units with breaks in between;
11:30 till 12:30: discussion with the lecturer;
15:00 till 17:30 work on homework exercises,
work in teams on problems posed by the lecturer,
presentation of results.

5. Grading

The basis for grading is provided by the performance of the students in the following items:

a) Homework exercises will be regularly given in order to achieve a better understanding of the lectures.

b) During each week of the course a project should be performed by the students. They should work in teams of 4 – 5 persons. The results obtained have to be presented by the teams.

c) An oral examination is planned to take place. It can consist of several parts taken during the course.

In order to obtain the grade for the course the following weights will be used for the items a) – c) from above:

Homework exercises	20%
Projects	50%
Oral examination	30%

The European Credit Transfer System (ECTS) is used for the grading of all performance assessments.